**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Sol**: - The work begin after 10 min, so the average time increase from 45min to 55min.

Normal distribution:-

= (X-μ)/ *σ*

= (60-55)/8

= 0.625

In R software for probability finding we use function called pnorm

As we want to find the probability of service manager cannot meet his commitment, so we should write below command.

1-pnorm (0.625)

= 0.2659855

The answer is **B. 0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Sol**: - Mean (μ) = 38

Standard deviation σ = 6

Z score = (Value - Mean)/SD

1. Z score for 44 = (44 - 38)/6 = 1

Probability of x < 44 = 0.84134 = 84.134%

People above 44 age = 100 - 84.13 = 15.87%

= 63 out of 400

Z score for 38 = (38 - 38)/6 = 0 = 0.5 = 50%

People between 38 and 44 age = 84.134 – 50 = 34.134%

= 137 out of 400

More employees at the processing center are older than 44 than between 38 and 44. - **FALSE**

1. Z score for 30  = (30 - 38)/6 =  -1.33  =  0.091211 = 9.12%

= 36 out of 400

A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2X1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Sol**: -X1 ~ N (μ, σ2) and X2 ~ N (μ, σ2) are *iid* normal random variables

According to the *Central Limit Theorem*, any large sum of *independent, identically distributed* (iid) random variables is approximately Normal.

The Normal distribution is defined by two parameters, the mean,https://tex.z-dn.net/?f=%5Cmu and the variance, σ2 and written asX ~ N (μ, σ2).

  X1 ~ N (μ, σ2) and X2 ~ N (μ, σ2) are two independent identically distributed random variables.

From the properties of **normal random variables**,

If X ~ N (μ1, σ12) and Y ~ N (μ2, σ22).are two independent identically distributed random variables then: -

The **sum** of normal random variables is given by

## X + Y ~ N (µ1 + µ2, σ12 + σ22).

The **product** of X is given by

Z ~ N (aµ1, a2 σ12)

2X1 : -

Thus, following the property of product, we get

2X1 ~ N (2µ, 22σ2)

= 2X1 ~ N (2µ, 4σ2)

X1 + X2 : -

X1 + X2 ~ N (µ + µ, σ2 + σ2) ~ N (2µ, 2σ2)

The difference between the 2X1 and X1 + X2

2X1 – (X1 + X2) ~ N (2µ - 2µ, 2 σ12 + 4 σ22) ~ N (0, 6σ2)

The mean of 2X1 and X1 + X2 is same but the var(σ2) of 2X1 is Two times more than the variance of X1 + X2.

The difference between the two says that the two given variables are identically, independently distributed.

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Sol**: - p (a < x < b) = 0.99

Mean = 100

Standard Deviation = 20

By Using Python

stats**.**norm**.**interval (0.99, 100, 20)

= (48.48341392902199, 151.516586070978)

**D) 48.5, 151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N (5, 32) and Profit2 ~ N (7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Sol**: - Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = (5 + 7)\*45 = 540 Million

SD = sqrt (9 + 16) \* 45 = 225

1. 95% probability for the annual profit of the company is

stats.norm.interval (0.95, 540, 225)

(99.00810347848784, 980.9918965215122)

1. To compute 5th Percentile, we use the formula X=μ + Z\*σ; wherein from z table, 5 percentile = -1.645

540 + (-1.645) \* 225

= 169.875

The 5th percentile of profit (in Rupees) for the company = 169.875

1. The two divisions has a larger probability of making a loss in a given year P(x<0)

First division probability of making loss

Stats.norm.cdf (0, 5, 3)

= 0.0477903522728147

= 4.8%

Second division probability of making loss

Stats.norm.cdf (0, 7, 4)

= 0.040059156863817086

= 4%

First division has a more probability for making more loss